The greater the magnetic field intensity, the shorter the subsonic segment. By assigning a value to l, we can determine the value of H which will assure the transition through the speed of sound, and we can also find the intensity of the corresponding electrical field E from the following equations:

$$H^2 = \frac{k+1}{2k}\frac{Gc^2}{S\sigma l}\ln\frac{u - u_0}{u_3 - u_1}$$

at (17)

$$x = l \qquad E = \frac{k}{k-1}u_1\frac{H}{c}$$

Substituting (17) into (15) we get

$$u = u_3 - (u_3 - u_0)\left(\frac{u_3 - u_0}{u_3 - u_1}\right)^{-\frac{x}{l}} \tag{18}$$

The values $x < l$ correspond to the subsonic portion of the nozzle, whereas the values $x > l$ correspond to the supersonic portion. The maximum velocity u_3 is attained at infinity. At a given terminal velocity u_-, the overall length of the nozzle is

$$L = l\,\frac{\ln[(u_3 - u_0)/u_3 - u_1)]}{\ln[(u_3 - u_0)/u_3 - u_1]} \tag{19}$$

The temperature in the critical cross section is independent of the location of the section

$$T^* = u_1{}^2/kR \tag{20}$$

The maximum Mach number (at infinity), as well as the maximum temperature, are found from Eqs. (13) at $u = u_3$:

$$M_\infty = \sqrt{\frac{2k}{k-1}}$$

$$T_\infty = \frac{u_1{}^2}{2(k-1)R} = \frac{k}{2(k-1)}T^* \tag{21}$$

When accelerating the supersonic stream ($M_0 > 1$), in particular at $u_1 = u_1{}^*$, the intensity of the magnetic field is determined by Eq. (12) from the known value of the thermal velocity u_- and length of the accelerating section $x = L$:

$$H^2 = \frac{k+1}{2k}\frac{G}{S}\frac{c^2}{\sigma L}\ln\frac{u_3 - u_0}{u_3 - u_-} \tag{22}$$

The velocity along the channel in this case is given by

$$u = u_3 - (u_3 - u_0)\left(\frac{u_3 - u_0}{u_3 - u_-}\right)^{-x/L} \tag{23}$$

Integrating Ohm's law with respect to the length and using the velocity equation (15), we can obtain expressions for the current flowing through the gas and for the electric power consumed in acceleration:

$$I = \frac{k+1}{2k}\frac{G}{b}\frac{c}{H}(u - u_0)$$

$$N = \frac{k+1}{2k}u_3G(u - u_0) \tag{24}$$

where b is the electrode gap.

The author is grateful to V. A. Bashkatov for guiding the research.

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Influence of the Earth's Orbital Motion on Radar Measurements of Range and Velocity in Space

V. M. Vakhnin

General Propositions

When we measure the range (the distance from the earth) and velocity relative to the earth of a target in space at extreme distances (of the order of one or more astronomical units), we encounter a number of specific effects which become negligible in other cases. The propagation time of radio waves in these conditions reaches values of hundreds or thousands of seconds; in this time the measuring station moves tens of thousands of kilometers with the earth in its orbit. The earth's orbital velocity is about 10^{-4} the velocity of light, so that the effects of the special theory of relativity become noticeable. The presence of the interplanetary (and interstellar) medium influences the velocity of propagation of radio waves: given a path of the order of 1 a.u. or more this can create an appreciable error in determining distance, since we do not know just how much the velocity of propagation of radio waves differs from the velocity of light in a vacuum. In the present paper the writer considers the influence of the earth's orbital motion on the results of radar measurements of velocity and range.

We shall take as our initial values τ, the time delay of a radio signal in its path from the transmitting station to the target to the receiving station, and dT/dt, its time derivative, choosing these values because they are the ones to which we can reduce any actual parameters received from radar stations (delay time, phase delay, Doppler frequency, etc.).

To analyze the influence of the earth's orbital motion we shall find it convenient to consider the process of measurement in an inertial system of coordinates that is stationary in

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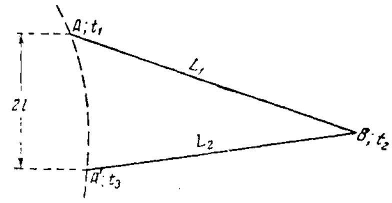
Fig. 1.



relation to the sun (Fig. 1). Figure 1 shows in diagrammatic form the relative positions of a target B and the earth at the time of transmission of a signal (point A) and at the time of reception of the reflected signal (point A').

The radio signal, emitted at the time t_1, is propagated in the direction of the target at a velocity v_s, which for purposes of the analysis of the geometric factors can be assumed to be known, constant, and equal to the velocity of light in a vacuum; at the time t_2 the signal is reflected (or relayed), and at the time t_3 the signal returns to the starting point. During the time the signal is travelling the measuring station moves in space a distance of

$$2l = v_E(t_3 - t_2) = \tau v_E \quad (1)$$

where v_E is the average velocity (during the interval τ) of the measuring point, made up of components representing the earth's orbital and rotational motion. By measuring the time delay τ we arrive at the complete path of the radio signal:

$$L_0 = L_1 + L_2 = \tau v_s \quad (2)$$

On the basis of the measurement of L_0 we can say that the observed target is located at an arbitrary point on the "surface of possible positions of the target" S_0, defined by the equation

$$L_1 + L_2 = \text{const} \quad (3)$$

According to (3), the surface S_0 is an ellipsoid of rotation with two foci, at points A and A'. The semimajor axis of the ellipsoid is equal to

$$a = (L_1 + L_2)/2 = L_0/2 \quad (4)$$

The eccentricity of the ellipsoid is equal to

$$e = 1/a = 2l/L_0 \quad (5)$$

Substituting Eqs. (1) and (2) into (5) we obtain for the eccentricity the formula

$$e = v_E/v_S \quad (6)$$

It is essential to note that, strictly speaking, the time t_2 is not determined uniquely but depends on the position of the target on the surface S_0.

Let us now look at the result of the measurement of the value $d\tau/dt$. It can be thought of as the result of comparing two measurements of the complete path L_0, separated in time by the short interval dt_3. Each of the measurements defines an ellipsoid of rotation described by formulas (3),(4), and (5). The two ellipsoids S_0 and S_0' are separated from each other both because of the values measured for the complete path L_0 and also because the observing point has moved in the time interval between observations (Fig. 2).

It is evident from Fig. 2 that in the "solar" system of coordinates we have used, the measurement of $d\tau/dt$ determines the component (v_1) of the velocity of the observed target perpendicular to the surface S_0. (The surface S_0' is assumed to be infinitely close to S_0.) The other components of velocity remain undetermined. The velocity v_1 relative to the "solar" coordinates is equal to

$$v_1 = \delta/dt_2 \quad (7)$$

where δ is the separation of the ellipsoids S_0 and S_0'.

The measurement of $d\tau/dt$ is made at a time t_3, so that it gives in fact the value of $d\tau/dt_3$. Increments dt_2 and dt_3 are

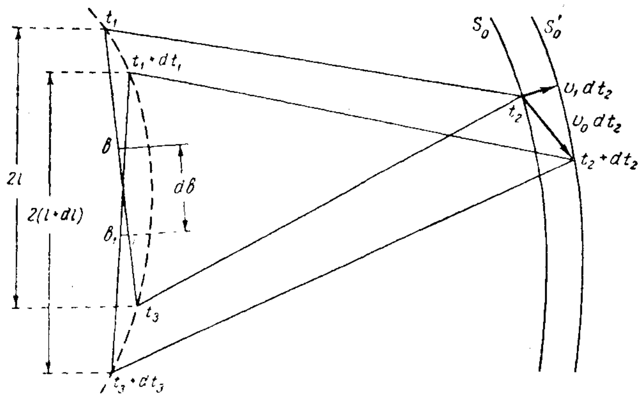


Fig. 2.

not in general equal, and this fact must be taken into account in analyzing the results.

The value of the separation δ is determined by three factors:

a) An increase in the dimensions of the ellipsoid resulting from an increase in the complete path by the value

$$dL_0 = v_S d\tau = v_S \frac{\partial \tau}{\partial t} dt \quad (8)$$

b) A displacement of the center of the ellipsoid (b) by the value

$$db = v_E[(dt_1 + dt_3)/2] \quad (9)$$

c) A rotation of the major axis of the ellipsoid by the angle

$$d\alpha = \Omega_E[(dt_1 + dt_3)/2] \quad (10)$$

where Ω_E is the angular velocity of the earth's motion around the sun.

The line bisecting the angle between paths L_1 and L_2 is normal to the surface of the ellipsoid of rotation, so that the displacement of the surface of the ellipsoid resulting from an increase in its dimensions is equal to

$$\delta_1 = (dL_0/2) \cos\gamma \quad (11)$$

where γ is the angle between a normal to the surface S_0 and the path L_1 or L_2.

The displacement resulting from the shift of the center of the ellipsoid is equal to

$$\delta_2 = db = v_E dt_2 \cos\epsilon \quad (12)$$

where ϵ is the angle between the direction of v_E and a normal to the surface S.

The displacement resulting from the rotation of the ellipsoid is equal to

$$\delta_3 = R \sin\zeta d\alpha \quad (13)$$

where R is the distance from the center of the ellipsoid to the target, and ζ is the angle between the direction to the target from the center of the ellipsoid and a normal to its surface.

Between dt_2 and dt_3 there exists the relationship

$$\frac{dt_2}{dt_3} = \frac{1 - (v_E/v_s)\cos\eta}{1 - (v_0/v_s)\cos\vartheta} \quad (14)$$

where η is the angle between v_E and L_2, and ϑ is the angle between v_0 and L_2 (Fig. 3).

Using Eqs. (7), (11), (12), and (13), we obtain for v_1 the expression

$$v_1 = \tfrac{1}{2}(dL_0/dt_2)\cos\gamma + v_E\cos\epsilon + R\Omega_E\sin\zeta \quad (15)$$

Using (2) and (14) and simplifying the expression by ignoring small terms of high order, we can reduce (15) to the form

$$v_1 = \frac{1}{2} v_S \frac{d\tau}{dt_3} \frac{1 - (v_0/v_S)\cos\vartheta}{1 - (v_E/v_S)\cos\eta} + v_E\cos\epsilon + R\Omega_E\sin\zeta \quad (16)$$

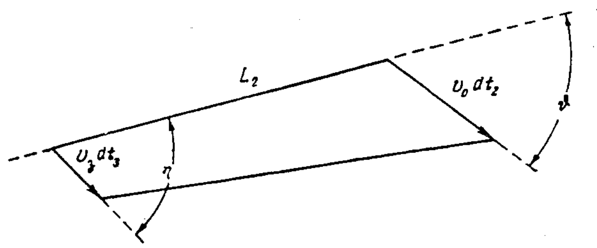


Fig. 3.

The first term in (16) is the usual Doppler component determined for the case when the measuring point is in motion. This term contains correction factors taking account of: a) the difference in the intervals dt_2 and dt_3; and b) the difference in the directions between a normal to S_0 and the radio signal beams. The second term enables us to convert the results from a measuring point in motion to the "solar" system of coordinates; this term takes into account only the straightforward rectilinear motion. The third term allows for the curvature of the trajectory of the measuring point. Numerical estimates of this term give the result

$$0 \leq (R\Omega_E \sin\zeta) \leq 10^{-11}R \tag{17}$$

Thus, for example, at a distance of 10^8 km the correction will be of the order of 1 m/sec.

Evaluation of Results

The motions of all points considered were determined relative to the "solar" system of coordinates; the propagation velocity of radio signals is determined relative to the same system of reference. Thus the system of reference can be considered stationary, and relativistic effects can therefore be assumed not to exist. Strictly speaking, however, to ignore relativistic effects we must use standards of length and time that are stationary in the "solar" system of coordinates, whereas the real measurements use "earth" standards moving in the solar system.

In order to preserve the rigor of our calculations we must, after computing motion in "solar" coordinates, introduce into the results of the measurements τ and $d\tau/dt$ a correction for the Lorentz time dilation

$$\tau_s = \frac{\tau_E}{\sqrt{1 - (v_E/v_S)^2}} \tag{18}$$

where τ_S is the delay measured in "solar" units of time and τ_E is the delay measured in "earth" units of time.

Here, strictly speaking, all mechanical characteristics of the solar system—the dimensions of orbits, velocities, periods of revolution, and gravitational potentials of celestial bodies—should also be measured in terms of "solar" scales of length and time.

It is interesting to note that, in astronomy, conversion from "earth" scales to "solar" scales, with the orbit considered in the "solar" system of coordinates, results in a proportional increase in all dimensions, velocities, and periods of revolution by the Lorentz factor

$$\frac{1}{\sqrt{1 - (v_E/v_S)^2}} \tag{19}$$

In reality, all measurements of the distances between celestial bodies made by radar and in relation to stationary "solar" coordinates are increased on conversion from the "earth" to the "solar" time scale by a proportional factor (19) which takes account of the presence of the Lorentz time dilation in "earth" scales. Goniometric measurements of distance made on earth in a direction perpendicular to the earth's motion and relative to solar coordinates must also be increased proportionally on conversion to a "solar" scale (19), so as to take into account the fact that on earth the standard of measurement is reduced by the Lorentz scale contraction.

The periods of revolution of celestial bodies, on transferring from "earth" to "solar" standards, increase proportionally as well (19); from the relationship $a^3 : T^2 = \mu : 4\pi$ (Kepler's third law) it follows that the gravitational potential is also increased proportionally by the same factor.

The relationships we have been considering are true only when the motion of celestial bodies is determined throughout in the inertial "solar" system of coordinates alone, and the influence of the earth's motion is included only as a correction to the results of the actual measurements.

If we take as a basis the inertial system of coordinates referred to the earth—more precisely, to the measuring point—the foregoing relationships take on a different form. Interpretation of the measurements must then be based on the postulate of the constancy of the velocity of light in a vacuum relative to the "earth" system of coordinates, and the simple relationships shown next will then be correct (to within a limit set by the effects of the influence of the interplanetary gas on the propagation of radio waves):

$$R_0 = L_0/2 = v_S\tau/2 \tag{20}$$

$$v_R = dR/dt = (v_S/2)(d\tau/dt) \tag{21}$$

where v_R is the radial component of velocity.

The time of measurement t_2 is also determined in this case with extreme simplicity:

$$t_2 = (t_1 + t_3)/2 \tag{22}$$

In this case the surface of the ellipsoid S_0 is transformed to a sphere with radius R_0. It is obvious that the results of Eq. (20) and the results of (4) and (5), determining the dimensions of S_0, are not contradictory. Actually, the ellipsoid S_0, when transferred to the "earth" system of coordinates, must be affected by the Lorentz contraction in the direction of the earth's motion. This transforms S_0 to a sphere with radius

$$R' = (L_0/2)\sqrt{1 - (v_E/v_S)^2} \tag{23}$$

Furthermore, in the "earth" system of coordinates there is no Lorentz time dilation such as existed when the earth was regarded as moving in relation to a chosen system of reference. This will require the increase of interval τ:

$$\tau = \tau' \frac{1}{\sqrt{1 - (v_E/v_S)^2}} \tag{24}$$

where τ is the interval of time in the "earth" system of coordinates. τ' is the same interval in the system of coordinates in which the earth moves. Taking Eqs. (23) and (24) into account, we see that ellipsoid S_0 in the "earth" coordinates should be represented by a sphere with radius

$$R'' = \frac{R'}{\sqrt{1 - (v_E/v_S)^2}} = R_0 \tag{25}$$

which confirms the agreement of (20) with the results obtained earlier in (4) and (5).

However, in determining the orbits of celestial bodies relative to inertial "earth" coordinates we must take into account the Lorentz contraction of dimensions, which does not remain invariable but changes with the change in the direction of the earth's motion around the sun. Therefore in the "earth" system of coordinates, the classical laws of celestial mechanics are precise only to the extent that the Lorentz factor is not taken into account. Also, the introduction of "earth" coordinates assumes that the earth's motion is rectilinear and uniform, which is true only approximately and only for short intervals of time. Thus in the case under consideration we cannot take into account the correction for the curvature of the earth's motion introduced in formula (17).

It should be noted that formulas (4), (6), and (16) differ from the very simple approximation formulas (20) and (21) only by the value of the relativistic correction (19). This correction is quite small and is commensurate with other errors of measurement: the error resulting from our ignorance of the exact value of the velocity of light in a vacuum and the correction for the influence of the interplanetary medium.

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Reviewer's Comment

Advancement in any field of scientific endeavor requires a continual review of pertinent parameters and the techniques used in applying them to specific problems. A prime example of this is the accuracy of radar measurements. Prior to the "space age," radars were required to provide information about objects at relatively short ranges with low velocities. As the state of the art progresses, not only do the target range and velocities increase, but also the accuracy with which they must be measured. In order to accomplish this, a greater number of sources of error must be considered, since those which were once considered secondary in nature are now significant. In V. M. Vakhnin's paper two sources of error are discussed which are not commonly considered, that is, earth's motion and special relativity effects. This is not meant to imply that he is the first to consider these effects, for there are a number of papers in which the author either studies them to a certain degree or acknowledges their existence and suggests further study. However, an admittedly incomplete survey of the literature on the subject does indicate that Vakhnin's work is current enough to be a significant contribution. His paper is well written and straightforward with regard to the development of equations and results. There are, however, two apparent errors. The first, Eq. (1), should read $2l = v_E(t_3 - t_1) = v_E T$. The second occurs in the third term of Eqs. (15) and (16) which should contain a factor $\cos E$ based on a combination of Eqs. (9), (10), and (12). It should be emphasized that these are not errors in theory or development but mistakes either in copying or omissions.

The following three papers deal with essentially the same subjects and might be of interest for purposes of comparison and/or clarification. The first is: "Relativistic reaction systems and asymmetry of time scales" by K. I. Kowalski, with reference appearing in Astronautics Information, Open Literature Survey, California Institute of Technology: J. P. L., in which time dilation due to the asymmetry of time scales between inertial and noninertial reference frames is illustrated. The second paper is "Principles of Doppler inertial guidance" by Dworetzky and Edwards, contained in a Space Anthology by General Precision Incorporated, which includes some considerations of earth's angular velocity effects on radar measurements. The third paper, considered to be the most significant by virtue of its recent publication, is "Relativistic and classical Doppler electronic tracking accuracies" by J. Hoffman, presented in March 1963 at a symposium of the American Institute of Aeronautics and Astronautics.

—RICHARD A. ENSTROM

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Geomagnetic Effects of Explosions in the Lower Atmosphere

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This paper reports the results of work on the study of geomagnetic effects in the lower atmosphere (below 80 km), published in 1959–61. It is assumed that the initial changes of the field in the geomagnetic effect due to the explosion near Christmas Island were caused by the penetration through the ionosphere of a shock wave formed as a result of that explosion, whereas the shock waves of the explosions over Johnston Island induced a strengthening of the field upon penetration through the F layer of the ionosphere. The delay time of the variation after the explosion near Christmas Island was equal to the time of penetration of the shock wave from the location of the explosion (about 10^6 cm over the earth's surface) to the E layer of the ionosphere. The speed of the shock wave is taken equal to about 3.3×10^4 cm/sec. The time of penetration of shock waves from the point of the explosions over Johnston Island to an altitude of about 200–300 km is fixed according to a formula for a point explosion in an inhomogeneous atmosphere, and turns out to be equal to 1–2 min for the explosion of August 1, 1958, and 2–9 min for the explosion of August 12, 1958, which agrees with the delay of positive impulses as observed, equal, respectively, to 2.0 and about 5 min.

THE first studies of the geomagnetic effects due to explosions in the lower atmosphere appeared in 1958. It was first considered in Ref. 1, in which the variation of the magnetic field of the earth was reported as observed in the magnetic observatory in Apia, after the nuclear explosion carried out by the United States in the atmosphere over the central part of the Pacific Ocean. The geomagnetic effects of this (August 1, 1958) and also the next (August 12, 1958) explosions over Johnston Island were studied in Refs. 2–10. The basic interest in these studies was directed toward the morphology

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